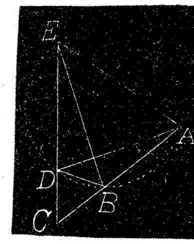
and on walking to B, a distance a, directly down the hill towards the foot of the pole the white part subtends the same angle. What is the length of the white part, if the point B is at a distance b from the foot of the pole?

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Applied Science, Texarkana College, Texarkana, Arkansas-Texas.

Let DE be the length painted white; then a circle will pass through A, B, D, E. Let $\angle EAD=\angle EBD=\alpha$, $AB=a$, $BC=b$, $\angle DAB=\angle DEB=\theta$, $\angle ABE=\angle ADE=\varphi$, $DC=y$, and $DE=x$.



Then $(x+y)y=(a+b)b$ ....(1).

$AE:a=\sin\varphi:\sin(\alpha+\theta+\varphi)$, $x:AE=\sin a:\sin\varphi$.

$$\therefore x=\frac{a\sin\alpha}{\sin(\alpha+\theta+\varphi)} \quad \ldots\ldots\ldots\ldots\ldots\ldots(2),$$

$b:x+y=\sin\theta:\sin(\alpha+\varphi)$ ....(3),

$(x+y):a+b=\sin(\alpha+\theta):\sin(a+\varphi)$ ....(4).

Eliminating θ between (3) and (4),

$$\left\{\frac{(x+y)^4}{(a+b)^2}-\frac{2b(x+y)^2\cos\alpha}{a+b}+b^2\right\}\sin^2(\alpha+\varphi)=(x+y)^2\sin^2\alpha \quad \ldots\ldots(5).$$

Eliminating θ between (2) and (3),

$$[\{b^2x^2-x^2(x+y)^2\}^2+4a^2b^2x^2(x+y)^2\sin^2\alpha]\sin^4(\alpha+\varphi)$$
$$-2a^2\sin^2\alpha(x+y)^2\{b^2x^2+x^2(x+y)^2\}$$
$$\sin^2(\alpha+\varphi)+a^4(x+y)^4\sin^4\alpha=0 \quad \ldots\ldots\ldots\ldots(6).$$

Eliminating $\sin(\alpha+\varphi)$ between (5) and (6) we get an equation in x and y which with (1) gives us the value of x.

Solved with result in terms of *EC* by *A. H. HOLMES*, and *FREDERICK R. HONEY*.

PROBLEMS.

58. Proposed by I. J. SCHWATT, Ph. D., Instructor in Mathematics, University of Pennsylvania, Philadelphia, Pennsylvania.

1. The point of intersection K_a' of the tangent drawn to the circumcircle about the triangle ABC at A and the side BC is harmonic conjugate to K_a with respect to BC. (K_a is the point where the symmedian line through A of the triangle ABC meets the side BC.)

2. The point K_a' is the center of the Apollonius circle passing through A of the triangle ABC.

3. Grebes point is on the line joining the middle point of any side of a triangle with the middle point of the altitude to this side.

59. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

Show that the tangent plane at any point of the surface $a^2x^2+b^2y^2+c^2z^2=2bcyz+2acxz+2abxy$ intersects the surface $ayz+bzx+cxy=0$ in two straight lines at right angles to one another.

CALCULUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

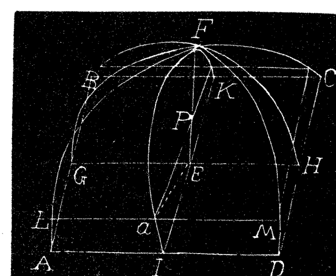
SOLUTIONS OF PROBLEMS.

47. Proposed by Prof. J. SCHEFFER, A. M., Hagerstown, Maryland.

The floor of a vault forms a square, and all sections parallel to it are squares. The two vertical sections through the middle points of the opposite sides of the floor are equal semi-circles. Find the convex surface and the volume of the vault.

I. Solution by C. W. M. BLACK, Professor of Mathematics in Wesleyan Academy, Wilbraham, Massachusetts; O. W. ANTHONY, M. Sc., Professor of Mathematics in New Windsor College, New Windsor, Maryland; A. H. HOLMES, Brunswick, Maine, and the PROPOSER.

Let $ABCD$ represent the base square, side$=2a$, and KEI and GFH the two equal semi-circles, radius$=a$. Let $LMNO$ be another square parallel to the base square, and at the distance $PE=x$ from it. The area of $LMNO$ is$=4(a^2-x^2)$,



$$\therefore \text{ Vol.}=4\int_0^a (a^2-x^2)dx=\tfrac{8}{3}a^3.$$

Denoting $\angle PEQ$ by θ, we have for the surface

$\int_0^{\frac{1}{2}\pi} 8a\cos\theta d(a\theta)=8a^2$. Or for the volume, $dV=4a^2\cos^2\theta dx$, where x is the vertical distance. $x=a\sin\theta$; $dx=a\cos\theta d\theta$.

$$\therefore\ V=4a^3\int_0^{\frac{1}{2}\pi}\cos^3\theta d\theta,\ =a^3\int_0^{\frac{1}{2}\pi}(\cos 3\theta+3\cos\theta)d\theta=\frac{8a^3}{3}.$$

II. Solution by BENJ. F. YANNEY, A. M., Professor of Mathematics in Mount Union College, Alliance, Ohio.

The convex surface of the vault is equivalent to the surface of a right cir-